Non-Markovian optimal stopping problems and constrained BSDEs with jump

Marco Fuhrman
Politecnico di Milano, Dipartimento di Matematica
via Bonardi 9, 20133 Milano, Italy
marco.fuhrman@polimi.it

Huyên Pham
LPMA - Université Paris Diderot
Batiment Sophie Germain, Case 7012
13 rue Albert Einstein, 75205 Paris Cedex 13
and CREST-ENSAE
pham@math.univ-paris-diderot.fr

Federica Zeni
Politecnico di Milano, Dipartimento di Matematica
via Bonardi 9, 20133 Milano, Italy
federica.zeni@mail.polimi.it

Abstract

We consider a non-Markovian optimal stopping problem on finite horizon. We prove that the value process can be represented by means of a backward stochastic differential equation (BSDE), defined on an enlarged probability space, containing a stochastic integral having a one-jump point process as integrator and an (unknown) process with a sign constraint as integrand. This provides an alternative representation with respect to the classical one given by a reflected BSDE. The connection between the two BSDEs is also clarified. Finally, we prove that the value of the optimal stopping problem is the same as the value of an auxiliary optimization problem where the intensity of the point process is controlled.

MSC Classification (2010): 60H10, 60G40, 93E20.

1 Introduction

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space and let $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ be the natural augmented filtration generated by an m-dimensional standard Brownian motion W. For given $T > 0$ we denote $L^2_T = L^2(\Omega, \mathcal{F}_T, \mathbb{P})$ and introduce the following spaces of processes.

1. $\mathcal{H}^2 = \{Z : \Omega \times [0,T] \to \mathbb{R}^m$, $\mathbb{F}$-predictable, $\|Z\|^2_{\mathcal{H}^2} = \mathbb{E}\int_0^T |Z_s|^2 ds < \infty\}$;
2. $\mathcal{S}^2 = \{Y : \Omega \times [0,T] \to \mathbb{R}$, $\mathbb{F}$-adapted and càdlàg, $\|Y\|^2_{\mathcal{S}^2} = \mathbb{E}\sup_{t\in[0,T]} |Y_s|^2 < \infty\}$;
3. $\mathcal{A}^2 = \{K \in \mathcal{S}^2$, $\mathbb{F}$-predictable, nondecreasing, $K_0 = 0\}$;
4. $\mathcal{S}^2_c = \{Y \in \mathcal{S}^2$ with continuous paths$\}$;

5. $\mathcal{A}_c^2 = \{K \in \mathcal{A}^2 \text{ with continuous paths}\}$.

We suppose we are given

$$f \in \mathcal{H}^2, \quad h \in \mathcal{S}_c^2, \quad \xi \in L_T^2, \quad \text{satisfying} \quad \xi \geq h_T. \tag{1.1}$$

We wish to characterize the process defined, for every $t \in [0,T]$, by

$$I_t = \operatorname*{ess\,sup}_{\tau \in \mathcal{T}_t(\mathbb{F})} \mathbb{E}\left[\int_t^{T\wedge\tau} f_s\, ds + h_\tau\, 1_{\tau<T} + \xi\, 1_{\tau \geq T} \,\middle|\, \mathcal{F}_t \right],$$

where $\mathcal{T}_t(\mathbb{F})$ denotes the set of $\mathbb{F}$-stopping times $\tau \geq t$. Thus, I is the value process of a non-Markovian optimal stopping problem with cost functions f, h, ξ. In [5] the process I is described by means of an associated reflected backward stochastic differential equation (BSDE), namely it is proved that there exists a unique $(Y, Z, K) \in \mathcal{S}_c^2 \times \mathcal{H}^2 \times \mathcal{A}_c^2$ such that, $\mathbb{P}$-a.s.

$$Y_t + \int_t^T Z_s\, dW_s \;=\; \xi + \int_t^T f_s\, ds + K_T - K_s, \tag{1.2}$$

$$Y_t \geq h_t, \qquad \int_0^T (Y_s - h_s)\, dK_s = 0, \qquad t \in [0,T], \tag{1.3}$$

and that, for every $t \in [0,T]$, we have $I_t = Y_t$ $\mathbb{P}$-a.s.

It is our purpose to present another representation of the process I by means of a different BSDE, defined on an enlarged probability space, containing a jump part and involving sign constraints. Besides its intrinsic interest, this result may lead to new methods for the numerical approximation of the value process, based on numerical schemes designed to approximate the solution to the modified BSDE. In the context of a classical Markovian optimal stopping problem, this may give rise to new computational methods for the corresponding variational inequality as studied in [2].

We use a randomization method, which consists in replacing the stopping time τ by a random variable η independent of the Brownian motion and in formulating an auxiliary optimization problem where we can control the intensity of the (single jump) point process $N_t = 1_{\eta \leq t}$. The auxiliary randomized problem turns out to have the same value process as the original one. This approach is in the same spirit as in [8], [9], [3], [4], [6] where BSDEs with barriers and optimization problems with switching, impulse control and continuous control were considered.

2 Statement of the main results

We are given $(\Omega, \mathcal{F}, \mathbb{P})$, $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$, W, T as before, as well as f, h, ξ satisfying (1.1). Let η be an exponentially distributed random variable with unit mean, defined in another probability space $(\Omega', \mathcal{F}', \mathbb{P}')$. Define $\bar\Omega = \Omega \times \Omega'$ and let $(\bar\Omega, \bar{\mathcal{F}}, \bar{\mathbb{P}})$ be the completion of $(\bar\Omega, \mathcal{F} \otimes \mathcal{F}', \mathbb{P} \otimes \mathbb{P}')$. All the random elements W, f, h, ξ, η have natural extensions to $\bar\Omega$, denoted by the same symbols. Define

$$N_t = 1_{\eta \leq t}, \qquad A_t = t \wedge \eta,$$

and let $\bar{\mathbb{F}} = (\bar{\mathcal{F}}_t)_{t\geq 0}$ be the $\bar{\mathbb{P}}$-augmented filtration generated by (W, N). Under $\bar{\mathbb{P}}$, A is the $\bar{\mathbb{F}}$-compensator (i.e., the dual predictable projection) of N, W is an $\bar{\mathbb{F}}$-Brownian motion independent of N and (1.1) still holds provided $\mathcal{H}^2$, $\mathcal{S}_c^2$, L_T^2 (as well as $\mathcal{A}^2$ etc.) are understood with respect to $(\bar\Omega, \bar{\mathcal{F}}, \bar{\mathbb{P}})$ and $\bar{\mathbb{F}}$ as we will do. We also define

$$\mathcal{L}^2 = \{U : \bar\Omega \times [0,T] \to \mathbb{R},\ \bar{\mathbb{F}}\text{-predictable}, \quad \|U\|_{\mathcal{L}^2}^2 = \bar{\mathbb{E}} \int_0^T |U_s|^2 dA_s = \bar{\mathbb{E}} \int_0^T |U_s|^2 dN_s < \infty\}.$$

We will consider the BSDE

$$\bar{Y}_t + \int_t^T \bar{Z}_s\, dW_s + \int_{(t,T]} \bar{U}_s\, dN_s = \xi\, 1_{\eta \geq T} + \int_t^T f_s\, 1_{[0,\eta]}(s)\, ds + \int_{(t,T]} h_s\, dN_s + \bar{K}_T - \bar{K}_t, \quad t \in [0,T], \tag{2.4}$$

with the constraint

$$U_t \leq 0, \qquad dA_t(\bar{\omega})\, \bar{\mathbb{P}}(d\bar{\omega}) - a.s. \tag{2.5}$$

We say that a quadruple $(\bar{Y}, \bar{Z}, \bar{U}, \bar{K})$ is a solution to this BSDE if it belongs to $\mathcal{S}^2 \times \mathcal{H}^2 \times \mathcal{L}^2 \times \mathcal{A}^2$, (2.4) holds $\bar{\mathbb{P}}$-a.s., and (2.5) is satisfied. We say that $(\bar{Y}, \bar{Z}, \bar{U}, \bar{K})$ is minimal if for any other solution $(\bar{Y}', \bar{Z}', \bar{U}', \bar{K}')$ we have, $\bar{\mathbb{P}}$-a.s, $\bar{Y}_t \leq \bar{Y}'_t$ for all $t \in [0,T]$.

Our first main result shows the existence of a minimal solution to the BSDE with sign constraint and makes the connection with reflected BSDEs.

Theorem 2.1 *Under* (1.1) *there exists a unique minimal solution* $(\bar{Y}, \bar{Z}, \bar{U}, \bar{K})$ *to* (2.4)-(2.5)*. It can be defined starting from the solution* (Y, Z, K) *to the reflected BSDE* (1.2)-(1.3) *and setting, for* $\bar{\omega} = (\omega, \omega')$*,* $t \in [0,T]$*,*

$$\bar{Y}_t(\bar{\omega}) = Y_t(\omega) 1_{t < \eta(\omega')}, \qquad \bar{Z}_t(\bar{\omega}) = Z_t(\omega) 1_{t \leq \eta(\omega')}, \tag{2.6}$$

$$\bar{U}_t(\bar{\omega}) = (h_t(\omega) - Y_t(\omega)) 1_{t \leq \eta(\omega')}, \qquad \bar{K}_t(\bar{\omega}) = K_{t \wedge \eta(\omega')}(\omega). \tag{2.7}$$

Now we formulate an auxiliary optimization problem. Let $\mathcal{V} = \{\nu : \bar{\Omega} \times [0,\infty) \to (0,\infty),$ $\bar{\mathbb{F}}$-predictable and bounded$\}$. For $\nu \in \mathcal{V}$ define

$$L_t^\nu = \exp\left(\int_0^t (1 - \nu_s)\, dA_s + \int_0^t \log \nu_s\, dN_s \right) = \exp\left(\int_0^{t \wedge \eta} (1 - \nu_s)\, ds \right) (1_{t < \eta} + \nu_\eta 1_{t \geq \eta}).$$

Since ν is bounded, L^ν is an $\bar{\mathbb{F}}$-martingale on $[0,T]$ under $\bar{\mathbb{P}}$ and we can define an equivalent probability $\bar{\mathbb{P}}_\nu$ on $(\bar{\Omega}, \bar{\mathcal{F}})$ setting $\bar{\mathbb{P}}_\nu(d\bar{\omega}) = L_t^\nu(\bar{\omega})\, \bar{\mathbb{P}}(d\bar{\omega})$. By a theorem of Girsanov type (Theorem 4.5 in [7]) on $[0,T]$ the $\bar{\mathbb{F}}$-compensator of N under $\bar{\mathbb{P}}_\nu$ is $\int_0^t \nu_s\, dA_s$, $t \in [0,T]$, and W remains a Brownian motion under $\bar{\mathbb{P}}_\nu$. We wish to characterize the value process J defined, for every $t \in [0,T]$, by

$$J_t = \operatorname*{ess\,sup}_{\nu \in \mathcal{V}} \bar{\mathbb{E}}_\nu \left[\int_{t \wedge \eta}^{T \wedge \eta} f_s\, ds + h_\eta\, 1_{t < \eta < T} + \xi\, 1_{\eta \geq T} \,\middle|\, \bar{\mathcal{F}}_t \right]. \tag{2.8}$$

Our second result provides a dual representation in terms of control intensity of the minimal solution to the BSDE with sign constraint.

Theorem 2.2 *Under* (1.1)*, let* $(\bar{Y}, \bar{Z}, \bar{U}, \bar{K})$ *be the minimal solution to* (2.4)-(2.5)*. Then, for every* $t \in [0,T]$*, we have* $\bar{Y}_t = J_t$ $\bar{\mathbb{P}}$*-a.s.*

The equalities $J_0 = \bar{Y}_0 = Y_0 = I_0$ immediately give the following corollary.

Corollary 2.1 *Under* (1.1)*, let* $(\bar{Y}, \bar{Z}, \bar{U}, \bar{K})$ *be the minimal solution to* (2.4)-(2.5)*. Then*

$$\bar{Y}_0 = \sup_{\tau \in \mathcal{T}_0(\mathbb{F})} \mathbb{E} \left[\int_0^{T \wedge \tau} f_s\, ds + h_\tau\, 1_{\tau < T} + \xi\, 1_{\tau \geq T} \right] = \sup_{\nu \in \mathcal{V}} \bar{\mathbb{E}}_\nu \left[\int_0^{T \wedge \eta} f_s\, ds + h_\eta\, 1_{\eta < T} + \xi\, 1_{\eta \geq T} \right].$$

3 Proofs

Proof of Theorem 2.1. Uniqueness of the minimal solution is not difficult and it is established as in [9], Remark 2.1.

Let $(Y,Z,K)\in\mathcal{S}^2_c\times\mathcal{H}^2\times\mathcal{A}^2_c$ be the solution to (1.2)-(1.3), and let $(\bar Y,\bar Z,\bar U,\bar K)$ be defined by (2.6), (2.7). Clearly it belongs to $\mathcal{S}^2\times\mathcal{H}^2\times\mathcal{L}^2\times\mathcal{A}^2$ and the constraint (2.5) is satisfied due to the reflection inequality in (1.3). The fact that it satisfies equation (2.4) can be proved by direct substitution, by considering the three disjoint events $\{\eta>T\}$, $\{0\le t<\eta<T\}$, $\{0<\eta<T,\eta\le t\le T\}$, whose union is $\bar\Omega$, $\bar{\mathbb{P}}$-a.s.

Indeed, on $\{\eta>T\}$ we have $Z_s=\bar Z_s$ for every $s\in[0,T]$ and, by the local property of the stochastic integral, $\int_t^T\bar Z_s\,dW_s=\int_t^T Z_s\,dW_s$, $\bar{\mathbb{P}}$-a.s. and (2.4) reduces to (1.2).

On $\{0\le t<\eta<T\}$ (2.4) reduces to

$$\bar Y_t+\int_t^T\bar Z_s\,dW_s+\bar U_\eta=\int_t^\eta f_s\,ds+h_\eta+\bar K_T-\bar K_t,\qquad \bar{\mathbb{P}}-a.s.;$$

since $\int_t^T\bar Z_s\,dW_s=\int_t^\eta Z_s\,dW_s$ $\mathbb{P}$-a.s., $h_\eta-\bar U_\eta=Y_\eta$ and, on the set $\{0\le t<\eta<T\}$, $\bar Y_t=Y_t$ and $\bar K_T-\bar K_t=K_\eta-K_t$, this reduces to

$$Y_t+\int_t^\eta Z_s\,dW_s=\int_t^\eta f_s\,ds+Y_\eta+K_\eta-K_t,\qquad \bar{\mathbb{P}}-a.s.$$

which again holds by (1.2).

Finally, on $\{0<\eta<T,\eta\le t\le T\}$ the verification of (2.4) is trivial, so we have proved that $(\bar Y,\bar Z,\bar U,\bar K)$ is indeed a solution.

Its minimality property will be proved later. □

To proceed further we recall a result from [5]: for every integer $n\ge1$, let $(Y^n,Z^n)\in\mathcal{S}^2_c\times\mathcal{H}^2$ denote the unique solution to the penalized BSDE

$$Y^n_t+\int_t^T Z^n_s\,dW_s=\xi+\int_t^T f_s\,ds+n\int_t^T(Y^n_s-h_s)^-\,ds,\qquad t\in[0,T];\tag{3.9}$$

then, setting $K^n_t=n\int_0^t(Y^n_s-h_s)^-\,ds$, the triple (Y^n,Z^n,K^n) converges in $\mathcal{S}^2_c\times\mathcal{H}^2\times\mathcal{A}^2_c$ to the solution (Y,Z,K) to (1.2)-(1.3).

Define

$$\bar Y^n_t(\bar\omega)=Y^n_t(\omega)1_{t<\eta(\omega')},\quad \bar Z^n_t(\bar\omega)=Z^n_t(\omega)1_{t\le\eta(\omega')},\quad \bar U^n_t(\bar\omega)=(h_t(\omega)-Y^n_t(\omega))1_{t\le\eta(\omega')},$$

and note that $\bar Y^n\to\bar Y$ in $\mathcal{S}^2$.

Lemma 3.1 *$(\bar Y^n,\bar Z^n,\bar U^n)$ is the unique solution in $\mathcal{S}^2\times\mathcal{H}^2\times\mathcal{L}^2$ to the BSDE: $\bar{\mathbb{P}}$-a.s.,*

$$\begin{aligned}\bar Y^n_t+\int_t^T\bar Z^n_s\,dW_s+\int_{(t,T]}\bar U^n_s\,dN_s &= \xi\,1_{\eta\ge T}+\int_t^T f_s\,1_{[0,\eta]}(s)\,ds\\ &\quad+\int_{(t,T]}h_s\,dN_s+n\int_t^T(\bar U^n_s)^+1_{[0,\eta]}(s)\,ds,\qquad t\in[0,T].\end{aligned}\tag{3.10}$$

Proof. $(\bar Y^n,\bar Z^n,\bar U^n)$ belongs to $\mathcal{S}^2\times\mathcal{H}^2\times\mathcal{L}^2$ and, proceeding as in the proof of Theorem 2.1 above, one verifies by direct substitution that (3.10) holds, as a consequence of equation (3.9). The uniqueness (which is not needed in the sequel) follows from the results in [1]. □

We will identify $\bar{Y}^n$ with the value process of a penalized optimization problem. Let $\mathcal{V}_n$ denote the set of all $\nu \in \mathcal{V}$ taking values in $(0,n]$ and let us define (compare with (2.8))

$$J_t^n = \operatorname*{ess\,sup}_{\nu \in \mathcal{V}_n} \bar{\mathbb{E}}_\nu \left[\int_{t\wedge\eta}^{T\wedge\eta} f_s\, ds + h_\eta\, 1_{t<\eta<T} + \xi\, 1_{\eta\geq T} \,\middle|\, \bar{\mathcal{F}}_t \right]. \tag{3.11}$$

Lemma 3.2 *For every $t \in [0,T]$, we have $\bar{Y}_t^n = J_t^n$ $\bar{\mathbb{P}}$-a.s.*

Proof. We fix any $\nu \in \mathcal{V}_n$ and recall that, under the probability $\bar{\mathbb{P}}_\nu$, W is a Brownian motion and the compensator of N on $[0,T]$ is $\int_0^t \nu_s\, dA_s$, $t \in [0,T]$. Taking the conditional expectation given $\bar{\mathcal{F}}_t$ in (3.10) we obtain

$$\begin{aligned}
\bar{Y}_t^n + \bar{\mathbb{E}}_\nu \left[\int_{(t,T]} \bar{U}_s^n\, \nu_s\, dA_s \,\middle|\, \bar{\mathcal{F}}_t \right] &= \bar{\mathbb{E}}_\nu \left[\xi\, 1_{\eta\geq T} + \int_t^T f_s\, 1_{[0,\eta]}(s)\, ds + \int_{(t,T]} h_s\, dN_s \,\middle|\, \bar{\mathcal{F}}_t \right] \\
&\quad + \bar{\mathbb{E}}_\nu \left[n \int_t^T (\bar{U}_s^n)^+ 1_{[0,\eta]}(s)\, ds \,\middle|\, \bar{\mathcal{F}}_t \right].
\end{aligned}$$

We note that $\int_{(t,T]} h_s\, dN_s = h_\eta\, 1_{t<\eta\leq T} = h_\eta\, 1_{t<\eta<T}$ $\bar{\mathbb{P}}_\nu$-a.s., since $\eta \neq T$ $\bar{\mathbb{P}}$-a.s. and hence $\bar{\mathbb{P}}_\nu$-a.s. Since $dA_s = 1_{[0,\eta]}(s)\, ds$ we have

$$\bar{Y}_t^n = \bar{\mathbb{E}}_\nu \left[\xi\, 1_{\eta\geq T} + \int_{t\wedge\eta}^{T\wedge\eta} f_s\, ds + h_\eta\, 1_{t<\eta<T} \,\middle|\, \bar{\mathcal{F}}_t \right] + \bar{\mathbb{E}}_\nu \left[\int_t^T (n(\bar{U}_s^n)^+ - \bar{U}_s^n\, \nu_s) 1_{[0,\eta]}(s)\, ds \,\middle|\, \bar{\mathcal{F}}_t \right]. \tag{3.12}$$

Since $nU^+ - U\,\nu \geq 0$ for every real number U and every $\nu \in (0,n]$ we obtain

$$\bar{Y}_t^n \geq \bar{\mathbb{E}}_\nu \left[\xi\, 1_{\eta\geq T} + \int_{t\wedge\eta}^{T\wedge\eta} f_s\, ds + h_\eta\, 1_{t<\eta<T} \,\middle|\, \bar{\mathcal{F}}_t \right]$$

for arbitrary $\nu \in \mathcal{V}_n$, which implies $\bar{Y}_t^n \geq J_t^n$. On the other hand, setting $\nu_s^\epsilon = n\, 1_{\bar{U}_s^n>0} + \epsilon\, 1_{-1\leq\bar{U}_s^n\leq 0} - \epsilon\, (\bar{U}_s^n)^{-1}\, 1_{\bar{U}_s^n<-1}$, we have $\nu^\epsilon \in \mathcal{V}_n$ for $0<\epsilon\leq 1$ and $n(\bar{U}_s^n)^+ - \bar{U}_s^n\, \nu_s \leq \epsilon$. Choosing $\nu = \nu^\epsilon$ in (3.12) we obtain

$$\bar{Y}_t^n \leq \bar{\mathbb{E}}_{\nu^\epsilon} \left[\xi\, 1_{\eta\geq T} + \int_{t\wedge\eta}^{T\wedge\eta} f_s\, ds + h_\eta\, 1_{t<\eta<T} \,\middle|\, \bar{\mathcal{F}}_t \right] + \epsilon\, T \leq J_t^n + \epsilon\, T$$

and we have the desired conclusion. □

Proof of Theorem 2.2. Let $(\bar{Y}', \bar{Z}', \bar{U}', \bar{K}')$ be any (not necessarily minimal) solution to (2.4)-(2.5). Since $\bar{U}'$ is nonpositive and $\bar{K}'$ is nondecreasing we have

$$\bar{Y}_t' + \int_t^T \bar{Z}_s'\, dW_s \geq \xi\, 1_{\eta\geq T} + \int_t^T f_s\, 1_{[0,\eta]}(s)\, ds + \int_{(t,T]} h_s\, dN_s = \xi\, 1_{\eta\geq T} + \int_{t\wedge\eta}^{T\wedge\eta} f_s\, ds + h_\eta\, 1_{t<\eta\leq T}.$$

We fix any $\nu \in \mathcal{V}$ and recall that W is a Brownian motion under the probability $\bar{\mathbb{P}}_\nu$. Taking the conditional expectation given $\bar{\mathcal{F}}_t$ we obtain

$$\bar{Y}_t' \geq \bar{\mathbb{E}}_\nu \left[\xi\, 1_{\eta\geq T} + \int_{t\wedge\eta}^{T\wedge\eta} f_s\, ds + h_\eta\, 1_{t<\eta<T} \,\middle|\, \bar{\mathcal{F}}_t \right],$$

where we have used again the fact that $\eta \neq T$ $\bar{\mathbb{P}}$-a.s. and hence $\bar{\mathbb{P}}_\nu$-a.s. Since ν was arbitrary in $\mathcal{V}$ it follows that $\bar{Y}_t' \geq J_t$ and in particular $\bar{Y}_t \geq J_t$.

Next we prove the opposite inequality. Comparing (2.8) with (3.11), since $\mathcal{V}_n \subset \mathcal{V}$ it follows that $J_t^n \leq J_t$. By the previous lemma we deduce that $\bar{Y}_t^n \leq J_t$ and since $\bar{Y}^n \to \bar{Y}$ in $\mathcal{S}^2$ we conclude that $\bar{Y}_t \leq J_t$. □

Conclusion of the proof of Theorem 2.1. It remained to be shown that the solution $(\bar{Y}, \bar{Z}, \bar{U}, \bar{K})$ constructed above is minimal. Let $(\bar{Y}', \bar{Z}', \bar{U}', \bar{K}')$ be any other solution to (2.4)-(2.5). In the previous proof it was shown that, for every $t \in [0, T]$, $\bar{Y}_t' \geq J_t$ $\bar{\mathbb{P}}$-a.s. Since we know from Theorem 2.2 that $\bar{Y}_t = J_t$ we deduce that $\bar{Y}_t' \geq \bar{Y}_t$. Since both processes are càdlàg, this inequality holds for every t, up to a $\bar{\mathbb{P}}$-null set. □

References

[1] Becherer, D. Bounded solutions to backward SDE's with jumps for utility optimization and indifference hedging. Ann. Appl. Probab. 16 (2006), no. 4, 2027-2054.

[2] Bensoussan, A. and Lions, J.L. Applications des inéquations variationnelles en contrôle stochastique. Dunod, 1978.

[3] Elie, R. and Kharroubi I. Adding constraints to BSDEs with jumps: an alternative to multidimensional reflections. ESAIM: Probability and Statistics 18 (2014), 233-250.

[4] Elie, R. and Kharroubi, I. BSDE representations for optimal switching problems with controlled volatility. Stoch. Dyn. 14 (2014), 1450003 (15 pages).

[5] El Karoui, N.; Kapoudjian, C.; Pardoux, E.; Peng, S.; Quenez, M. C. Reflected solutions of backward SDE's, and related obstacle problems for PDE's. Ann. Probab. 25 (1997), no. 2, 702-737.

[6] Fuhrman, M. and Pham, H. Randomized and backward SDE representation for optimal control of non-markovian SDEs. Preprint arXiv:1310.6943, to appear on Ann. Appl. Probab.

[7] Jacod, J. Multivariate point processes: predictable projection, Radon-Nikodym derivatives, representation of martingales. Z. Wahrscheinlichkeitstheorie und Verw. Gebiete 31 (1975), 235-253.

[8] Kharroubi I., J. Ma, H. Pham, and J. Zhang. Backward SDEs with constrained jumps and Quasi-variational inequalities. Ann. Probab. 38 (2010), 794-840.

[9] Kharroubi, I. and Pham H. (2012). Feynman-Kac representation for Hamilton-Jacobi-Bellman IPDE. Preprint arXiv:1212.2000, to appear on Ann. Probab.